THE PEST CONTROL IN SYSTEMS WITH ONE PREY AND TWO PREDATORS

CONTROLUL DĂUNĂTORILOR ÎN SISTEME CU O PRADĂ ŞI DOI PRĂDĂTORI

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Abstract. *Organic agriculture imposed biological control, which uses the living organisms to suppress pest populations. Organisms in a cropping system interact in many ways — through competition. There are different approaches in regard the possibility of the modeling of these complex systems. Over the last decade, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators especially for biological systems. Many examples of biological synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies. In order to formulate the pest control in this work the synchronization of two Lotka–Volterra systems with three species, one prey and two predators is presented. The transient time until synchronization depends on initial conditions of two systems and on the control number.*

Key words: biological control, chaos, synchronization, prey, predator

Rezumat. *Agricultura organica impune controlul biologic care utilizeaza organismele vii pentru a micsora populatia daunatorilor. Un ecosistem agricol consta dintr-un ansamblu de relatii intre plante de cultura sau pomi, erbivore, pradatori, buruieni etc. Organismele dintr-un astfel de sistem interactioneaza pe diferite cai fiind in competitie. Exista diferite moduri de abordare privind posibilitatea de modelare a acestor sisteme complexe. In ultimele decade exista un interes considerabil in generalizarea conceptului de sincronizare pentru a include oscilatorii haotici cuplati, in special pentru sisteme biologice. In literatura au fost documentate multe astfel de exemple dar cunoasterea teoretică duce la lipsa de studii experimentale. Pentru a analiza controlul daunatorilor, in aceasta lucrare se studiaza sincronizarea a doua sisteme Lotka–Volterra cu o prada si doi pradatori. Timpul de tranzitie pana la sincronizare depinde de conditiile initiale si de numarul populatiilor de control.*

Cuvinte cheie: control biologic, haos, sincronizare, pradă, prădător

INTRODUCTION

The pest control is of great interest in agriculture domain because the pests have been the major factor that reduces the agricultural production in the world. Different methods have been used in the process of pest management, for instance, chemical pesticides, biological pesticides, computers, atomic energy etc. Of all methods, chemical pesticides seem to be a convenient and efficient one, because they can quickly kill a significant portion of a pest population. But synthetic chemical pesticides introduced and used widely on agricultural crops in order to control the agricultural pests represent a significant food safety risk. Organic agriculture imposed biological control, which uses the living organisms to suppress pest populations.

Generally, from mathematical viewpoint, biological control has been modeled as a two-species interaction. In this case, the prey-predator or host parasitoid models ignore many important factors such as interactions between another species of same ecosystem, interactions with environment, etc. Arneodo et al. [1], have demonstrated that one can obtain chaotic behaviour for three species. In a 1988 paper Samardzija and Greller [9] propose a two-predator, one prey generalization of the Lotka-Volterra problem into three dimensions. The synchronization of trajectories of two attractors of this modifed, three-dimensional Lotka-Volterra equation, was performed by John Costello [2] using the Kapitaniak method.

THEORY

Samardzija and Greller [9] proposed equations for a two-predator, one prey generalization of the Lotka-Volterra system as follows:

$$\begin{aligned} \dot{x} &= x - xy + Cx^2 - Azx^2 \\ \dot{y} &= -y + xy \\ \dot{z} &= -Bz + Azx^2 \end{aligned} \qquad (1)$$

Here x is the prey population, y and z are predator populations and A,B, C are positive constants.

For A=2. 9851, B=3 and C=2 the systems has chaotic behaviour. For initial conditions x_0=1, y_0=1,4 şi z_0=1 the strange attractor is given in figure 1.

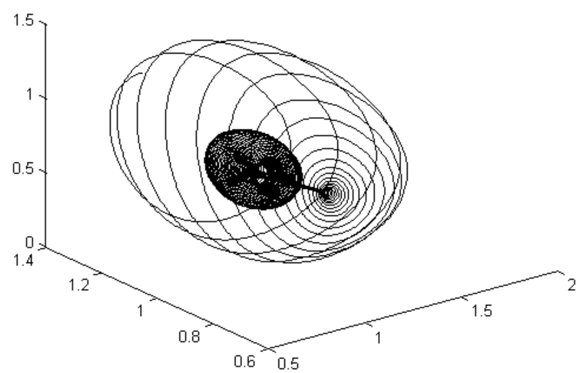

Fig. 1. Phase portrait of (x, y, z) for Samardzija and Greller system with one prey and two predators

Over the last decade, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators especially for biological systems [3], [7-8]. To synchronize two Lotka –Volterra systems whit three species we used a simple method for chaos synchronization proposed in [4-6].

If the chaotic system (master) is:

$\dot{x} = f(x)$ where $x = (x_1, x_2, ...x_n) \in R_n$

$f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \to R^n$

The slave system is:

$\dot{y} = f(y) + \varepsilon(y - x)$

where the functions $\dot{\varepsilon}_i = -\lambda_i (y_i - x_i)^2$ and λ_i are positive constants

RESULTS AND DISCUSSIONS

The slave system for the system (1) is:

$$\begin{aligned} \dot{x}_1 &= x_1 - x_1 y_1 + 2x^2 - 2.9851 z_1 x_1^2 + \varepsilon_1 (x_1 - x) \\ \dot{y}_1 &= -y_1 + x_1 y_1 + \varepsilon_2 (y_1 - y) \\ \dot{z}_1 &= -3z_1 + 2.9851 z_1 x_1^2 + \varepsilon_3 (z_1 - z) \end{aligned} \tag{2}$$

The control strength is of the form:

$$\begin{aligned} \dot{\varepsilon}_1 &= -10(x_1 - x)^2 \\ \dot{\varepsilon}_2 &= -10(y_1 - y)^2 \\ \dot{\varepsilon}_3 &= -10(z_1 - z)^2 \end{aligned} \tag{3}$$

Fig.2, 3, 4 and 5 show the syncronization of the two Lotka –Volterra generalized systems (for one prey and two predators).

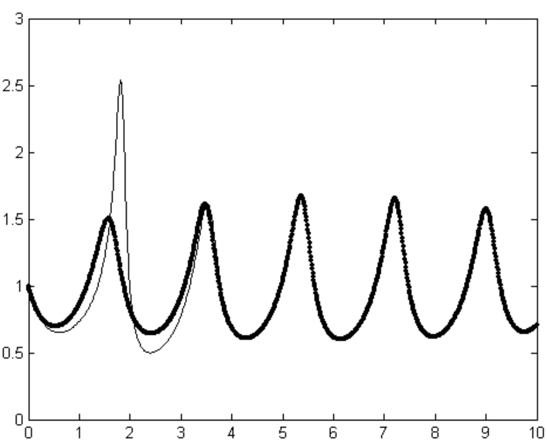

Fig.2 – x(t)-black, x_1(t)- gray [x(0) = 1; y(0) = 1.4 z(0) = 1; x_1(0) =1; y_1(0) = 1.5 ; z_1(0) = 1; ε_1(0)= ε_2 (0)= ε_3 (0) = 1]

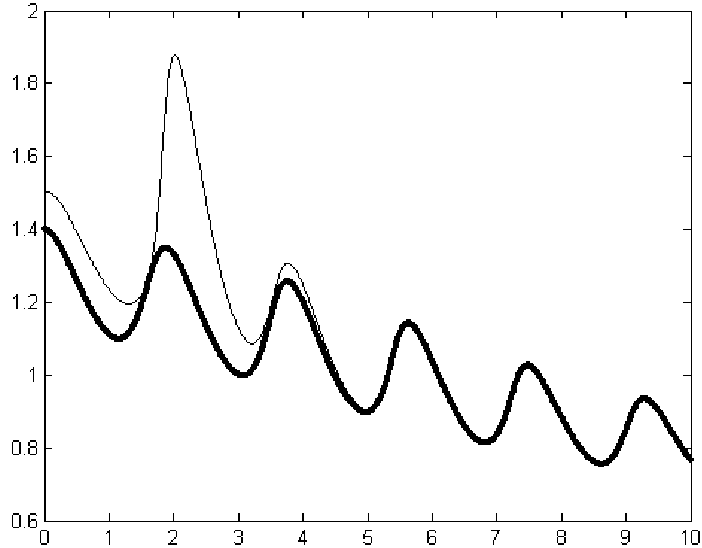

Fig. 3. y(t)-black, y_1(t)- gray [x(0) = 1; y(0) = 1.4 z(0) = 1; x_1(0) =1; y_1(0) = 1.5 ; z_1(0) = 1; $\varepsilon_1(0) = \varepsilon_2(0) = \varepsilon_3(0) = 1$]

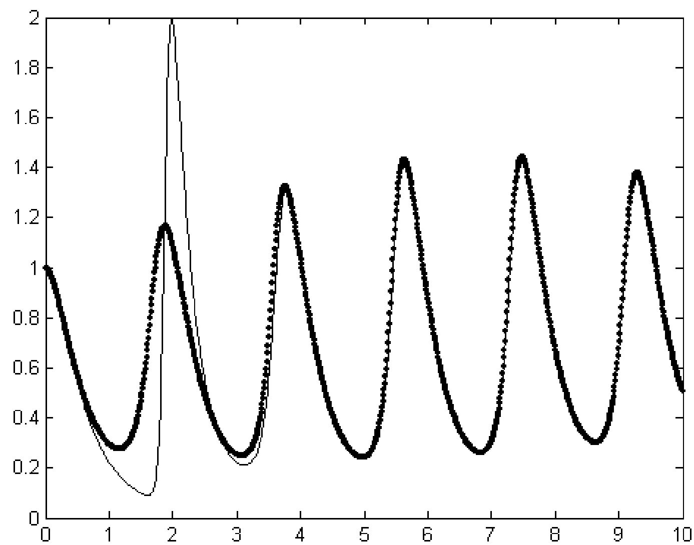

Fig. 4. z(t)-black, z_1(t)- gray [x(0) = 1; y(0) = 1.4 z(0) = 1; x_1(0) =1; y_1(0) = 1.5 ; z_1(0) = 1; $\varepsilon_1(0) = \varepsilon_2(0) = \varepsilon_3(0) = 1$]

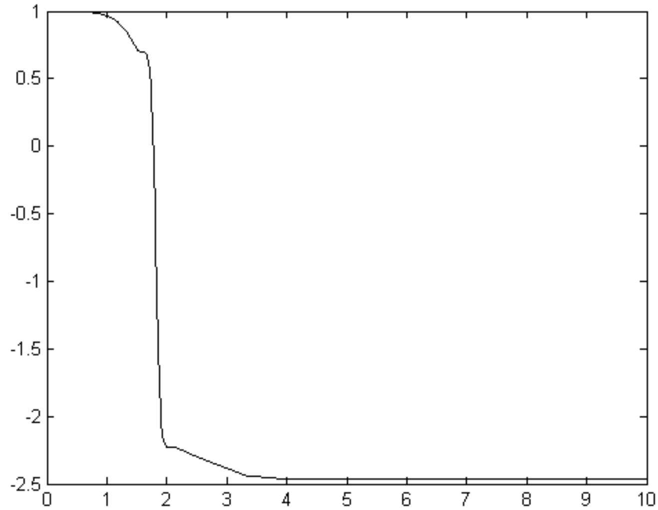

Fig. 5 – The control strength ε_1(t) [x(0) = 1; y(0) = 1.4 z(0) = 1; $x_1(0)$ =1; $y_1(0)$ = 1.5; $z_1(0)$ = 1; ε_1(0)= ε_2 (0)= ε_3 (0) = 1]

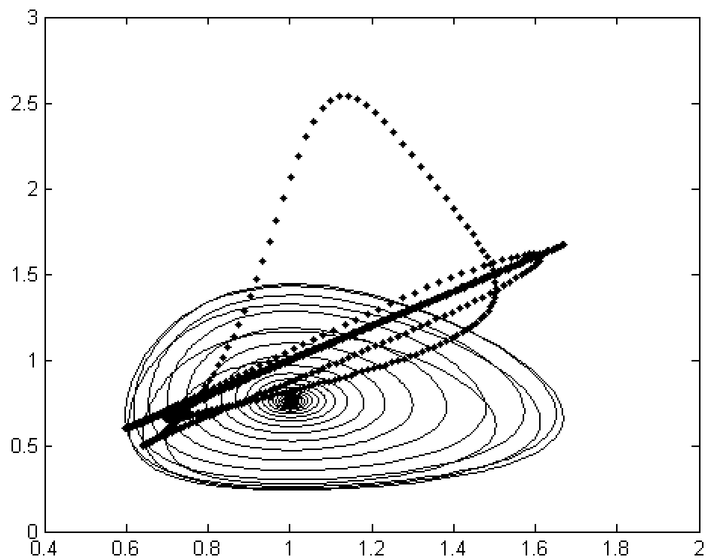

Fig. 6. Phase portrait of (x, z,) and (x,x_1) for Samardzija and Greller system with one prey and two predators [x(0) = 1; y(0) = 1.4 z(0) = 1; $x_1(0)$ =1; $y_1(0)$ = 1.5; $z_1(0)$ = 1; ε_1(0)= ε_2 (0)= ε_3 (0) = 1]

CONCLUSIONS

In order to formulate the biological control, the synchronization of two Lotka–Volterra systems with one prey and two predator is presented in this work. The transient time until synchronization depends on initial conditions of two systems and on the control strength. Therefore, we suggest that we can control the three species obtaining the synchronization of prey and predator population as a function of the control strength.

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